Geometry: 8.1-8.4 Notes

NAME______________________

<u>8.1 Similar Triangles</u>____ **Date:**________________

Define Vocabulary:

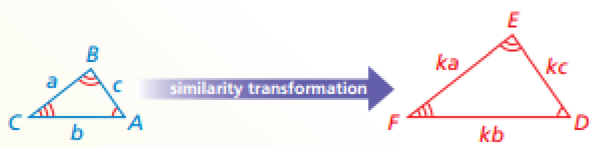
similar figures

similarity transformation

corresponding parts

Corresponding Parts of Similar Polygons

In the diagram below, $\triangle ABC$ is similar to $\triangle DEF$. You can write "$\triangle ABC$ is similar to $\triangle DEF$" as $\triangle ABC \sim \triangle DEF$. A similarity transformation preserves angle measure. So, corresponding angles are congruent. A similarity transformation also enlarges or reduces side lengths by a scale factor k. So, corresponding side lengths are proportional.



Corresponding angles

$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

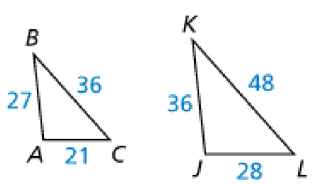
Ratios of corresponding side lengths

$\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA} = k$

Examples: Using similarity statements.

1. **<u>WE DO</u>**

In the diagram, $\triangle ABC \sim \triangle JKL$.

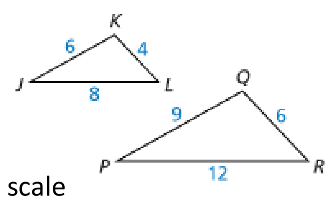


a. Find the scale factor from $\triangle ABC$ to $\triangle JKL$.

b. List all pairs of congruent angles.

c. Write the ratios of the corresponding side lengths in a *statement of proportionality*.

2. **<u>YOU DO</u>**



In the diagram, $\Delta JKL \sim \Delta PQR$. Find the scale factor from ΔJKL to ΔPQR. Then list all pairs of congruent angles and write the ratios of the corresponding side lengths in a statement of proportionality.

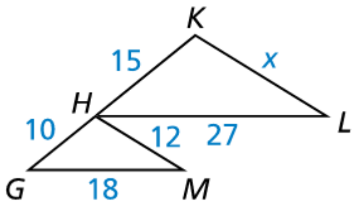
Corresponding Lengths in Similar Polygons

If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the scale factor of the similar polygons.

Examples: Finding a corresponding length.

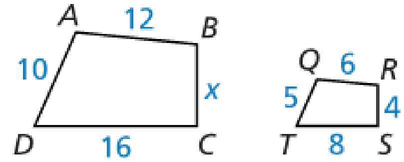
3. **WE DO**

In the diagram, $\triangle GHM \sim \triangle HKL$. Find the value of x.



4. **YOU DO**

Find the value of x.

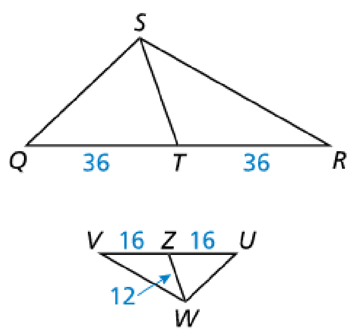


$ABCD \sim QRST$

Examples: Finding a corresponding length.

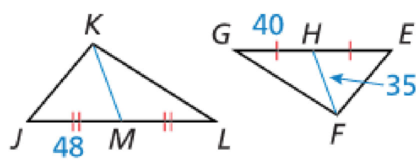
5. **WE DO**

In the diagram, $\triangle UVW \sim \triangle QRS$. Find the length of the median $\overline{ST}$.



6. **YOU DO**

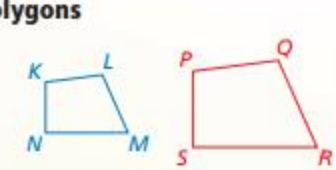
Find KM.



$\triangle JKL \sim \triangle EFG$

Theorem 8.1 Perimeters of Similar Polygons

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.



If $KLMN \sim PQRS$, then $\frac{PQ + QR + RS + SP}{KL + LM + MN + NK} = \frac{PQ}{KL} = \frac{QR}{LM} = \frac{RS}{MN} = \frac{SP}{NK}$.

Proof Ex. 52, p. 426; *BigIdeasMath.com*

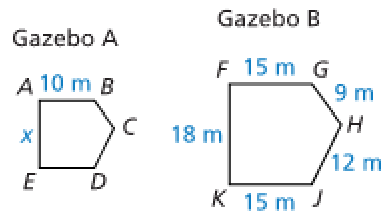
Examples: Modeling with mathematics

7. **WE DO**

Your neighbor has decided to enlarge his garden. The garden is rectangular with width 6 feet and length 15 feet. The new garden will be similar to the original one, but will have a length of 35 feet. Find the perimeter of the original garden and the enlarged garden.

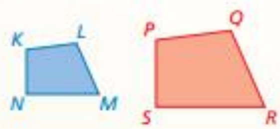
8. **YOU DO**

The two gazebos shown are similar pentagons. Find the perimeter of Gazebo A.



Theorem 8.2 Areas of Similar Polygons

If two polygons are similar, then the ratio of their areas is equal to the squares of the ratios of their corresponding side lengths.



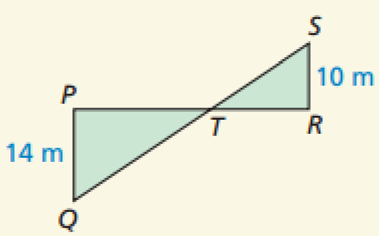
If $KLMN \sim PQRS$, then $\frac{\text{Area of } PQRS}{\text{Area of } KLMN} = \left(\frac{PQ}{KL}\right)^2 = \left(\frac{QR}{LM}\right)^2 = \left(\frac{RS}{MN}\right)^2 = \left(\frac{SP}{NK}\right)^2$.

Proof Ex. 53, p. 426; *BigIdeasMath.com*

Examples: Finding areas of similar polygons.

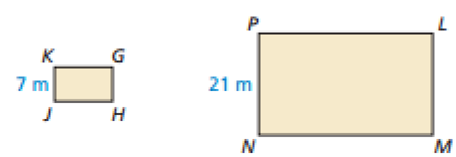
9. **WE DO**

In the diagram, $\triangle PQT \sim \triangle RST$, and the area of $\triangle RST$ is 75 square meters. Find the area of $\triangle PQT$.



10. **YOU DO**

In the diagram, $GHJK \sim LMNP$. Find the area of $LMNP$.



Area of $GHJK = 84 \text{ m}^2$

Assignment	

8.2 Proving Triangle similar by AA_____________ ____________________________ Date:_____________

Define Vocabulary:

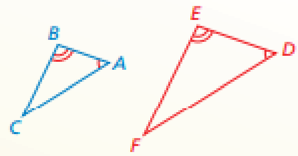
similar figures

similarity transformation

Theorem 8.3 Angle-Angle (AA) Similarity Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.

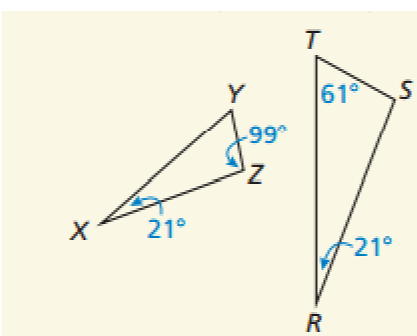
If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.



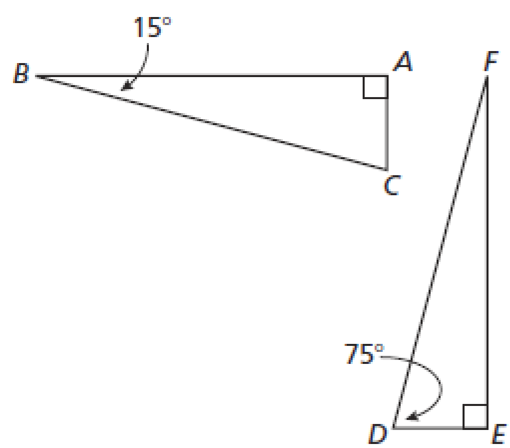
Proof p. 428

Examples: Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.

1. **WE DO**



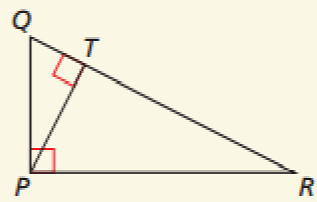
2. **YOU DO**



Examples: Show that the triangles are similar. Write a similarity statement.

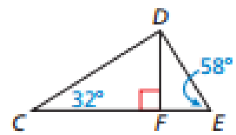
3. **WE DO**

Show that $\triangle QPR \sim \triangle QTP$.



4. **YOU DO**

$\triangle CDF$ and $\triangle DEF$



Examples: Modeling with mathematics

5. **WE DO**

A school flagpole casts a shadow that is 45 feet long. At the same time, a boy who is five feet eight inches tall casts a shadow that is 51 inches long. How tall is the flagpole to the nearest foot?

6. **YOU DO**

You are standing outside, and you measure the lengths of the shadows cast by both you and a tree. Write a proportion showing how you could find the height of the tree.

Assignment	

8.3 Proving triangle similarity with SSS and SAS.______________________________Date:_____________

Define Vocabulary:

corresponding parts

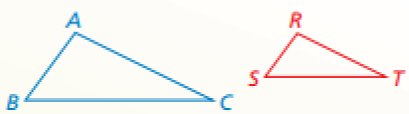
slope

parallel lines

perpendicular lines

Theorem 8.4 Side-Side-Side (SSS) Similarity Theorem

If the corresponding side lengths of two triangles are proportional, then the triangles are similar.



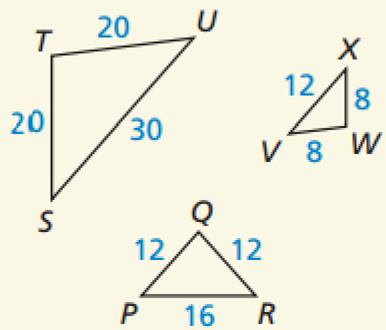
If $\frac{AB}{RS} = \frac{BC}{ST} = \frac{CA}{TR}$, then $\triangle ABC \sim \triangle RST$.

Proof p. 437

Examples: Using the SSS Similarity Theorem.

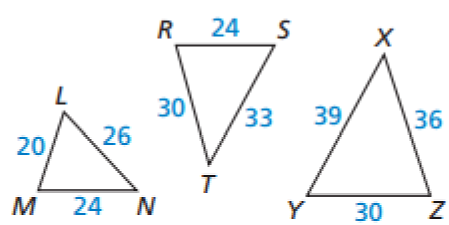
1. **WE DO**

Is either $\triangle PQR$ or $\triangle STU$ similar to $\triangle VWX$?



2. **YOU DO**

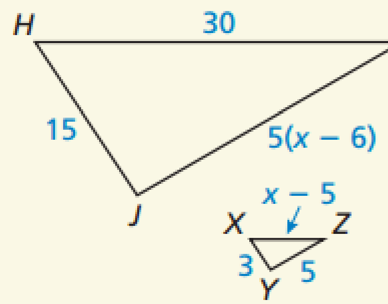
Which of the three triangles are similar? Write a similarity statement.



Examples: Using the SSS Similarity Theorem.

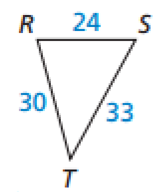
3. **WE DO**

Find the value of x that makes $\triangle XYZ \sim \triangle HJK$.



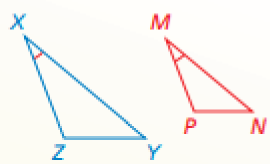
4. **YOU DO**

The shortest side of a triangle similar to $\triangle RST$ is 12 units long. Find the other side lengths of the triangle.



Theorem 8.5 Side-Angle-Side (SAS) Similarity Theorem

If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.



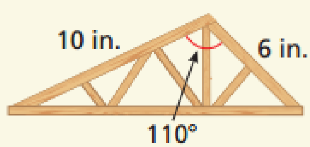
If $\angle X \cong \angle M$ and $\frac{ZX}{PM} = \frac{XY}{MN}$, then $\triangle XYZ \sim \triangle MNP$.

Proof Ex. 33, p. 443

Examples: Using the SAS Similarity Theorem.

5. **WE DO**

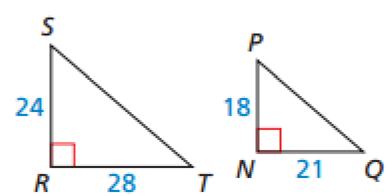
The diagram is a scale drawing of a triangular roof truss. The lengths of the two upper sides of the actual truss are 18 feet and 40 feet. The actual truss and the scale drawing both have an included angle of 110°. Is the scale drawing of the truss similar to the actual truss? Explain.



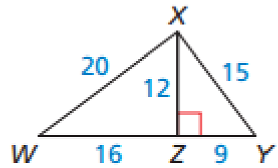
6. **YOU DO**

Explain how to show that the indicated triangles are similar.

$\triangle SRT \sim \triangle PNQ$

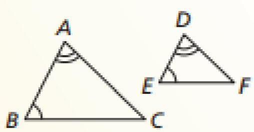


7. $\triangle XZW \sim \triangle YZX$



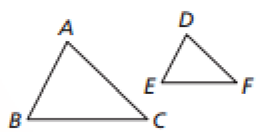
Triangle Similarity Theorems

AA Similarity Theorem



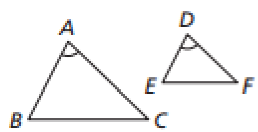
If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

SSS Similarity Theorem



If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

SAS Similarity Theorem



If $\angle A \cong \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

Assignment	

8.4 Proportionality Theorems Date:______________

Define Vocabulary:

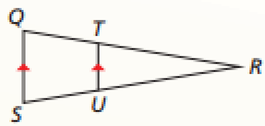
corresponding angles

ratio

proportion

Theorem 8.6 Triangle Proportionality Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

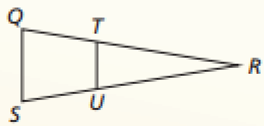


If $\overline{TU} \parallel \overline{QS}$, then $\frac{RT}{TQ} = \frac{RU}{US}$.

Proof Ex. 27, p. 451

Theorem 8.7 Converse of the Triangle Proportionality Theorem

If a line divides two sides of a triangle proportionally, then it is parallel to the third side.



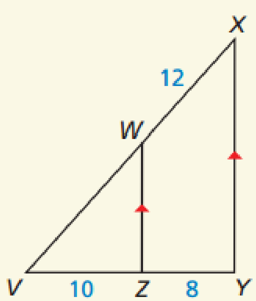
If $\frac{RT}{TQ} = \frac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$.

Proof Ex. 28, p. 451

Examples: Finding the length of a segment.

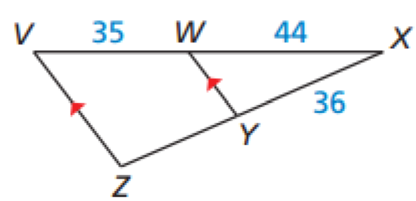
1. **WE DO**

In the diagram, $\overline{WZ} \parallel \overline{XY}$, $WX = 12$, $VZ = 10$, and $ZY = 8$. What is the length of $\overline{VW}$?



2. **YOU DO**

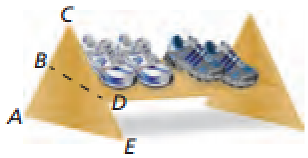
Find the length of $\overline{YZ}$.



Examples: Determining whether segments are parallel.

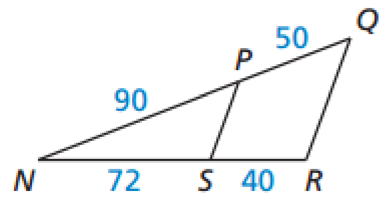
3. **WE DO**

BA = 35 centimeters,
CB = 25 centimeters,
CD = 20 centimeters, and
DE = 28 centimeters. Explain why the shelf is parallel to the floor.



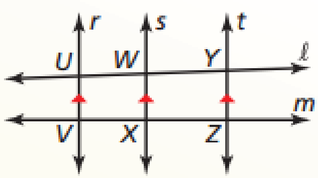
4. **YOU DO**

Determine whether $\overline{PS} \parallel \overline{QR}$.



Theorem 8.8 Three Parallel Lines Theorem

If three parallel lines intersect two transversals, then they divide the transversals proportionally.



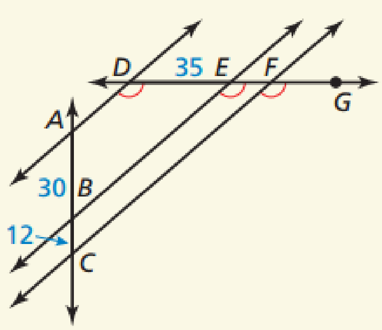
$$\frac{UW}{WY} = \frac{VX}{XZ}$$

Proof Ex. 32, p. 451

Examples: Using the Three Parallel Lines Theorem.

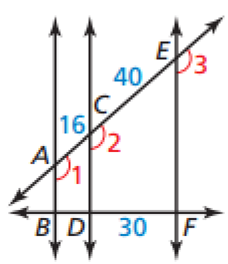
5. **WE DO**

In the diagram, ∠*ADE*, ∠*BEF*, and ∠*CFG* are all congruent. *AB* = 30, *BC* = 12, and *DE* = 35. Find *DF*.



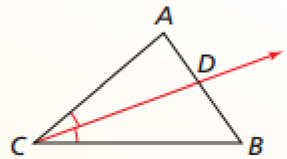
6. **YOU DO**

Find $\overline{BD}$.



Theorem 8.9 Triangle Angle Bisector Theorem

If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.



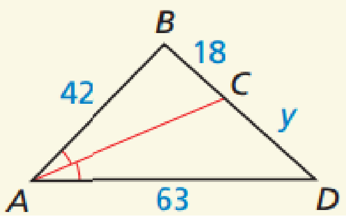
$$\frac{AD}{DB} = \frac{CA}{CB}$$

Proof Ex. 35, p. 452

Examples: Using the Three Parallel Lines Theorem.

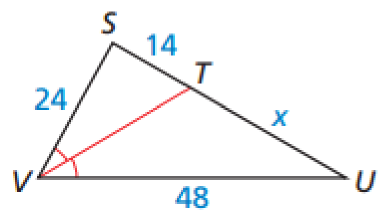
7. **WE DO**

In the diagram, $\angle BAC \cong \angle CAD$. Use the given lengths to find the length of $\overline{CD}$.



8. **YOU DO**

Find the value of the variable.



Assignment	